MODELING THE APPEARANCE OF THE ROUND BRILLIANT CUT DIAMOND: AN ANALYSIS OF FIRE, AND MORE ABOUT BRILLIANCE

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This article presents the latest results of GIA's research on the interaction of light with fully faceted colorless symmetrical round brilliant cut diamonds of various proportions. The second major article in this three-dimensional modeling study, it deals with fire—the visible extent of light dispersed into spectral colors. As fire is best seen with directed (spot) lighting, the metric for fire presented (dispersed colored light return, or DCLR) uses this lighting condition. DCLR values were computed for more than 26,000 combinations of round brilliant proportions. In general, different sets of proportions maximize DCLR and WLR (weighted light return, our metric for brilliance), but there are some proportion combinations that produce above-average values of both metrics. Analysis of these metric values with variations of five proportion parameters demonstrated that every facet contributes to the appearance of a round brilliant diamond. In particular, star and lower-girdle facet lengths—which are ignored by most cut-evaluation systems—could have a noticeable effect on WLR and DCLR. Observations of actual diamonds corroborate these results.

For more than 80 years, the diamond trade has debated which proportions produce the best-looking round brilliant (see, e.g., Ware, 1936; "Demand for ideal proportions . . . ," 1939; Dake, 1953; Liddicoat, 1957; Dengenhard, 1974; Eulitz, 1974), with discussions growing quite animated in the last decade (see, e.g., Boyajian, 1996; Kaplan, 1996; Gilbertson and Walters, 1996; Bates and Shor, 1999; Nestlebaum, 1999; Holloway, 2000). Many methods of evaluating cut have been presented, including several grading systems (see box A; also see table 3 in Hemphill et al., 1998). Although interest at GIA in how diamond cut relates to appearance extends back more than 50 years, we have been researching the topic using modern computer technology since 1989 (Manson, 1991; again, see Hemphill et al., 1998). Our overall research goal is to understand why a round brilliant cut diamond looks the way it does. Its appearance is a complex mixture of the effects of various lighting and observing conditions, the specific characteristics of each diamond, and the interpretation by the human visual system of the overall pattern of light shown by the diamond (figure 1). Traditionally, the appearance of the round brilliant diamond has been described using three aspects: brilliance, fire, and scintillation.

A method that scientists use to address a complicated problem is: (1) break it into simpler aspects, examining each aspect separately; and then (2) make sure that solutions for each small piece of the problem also hold true for the larger problem as a whole. We have applied this approach to our study of polished diamond appearance by examining each appearance aspect separately. In our report on the first of these (Hemphill et al., 1998), we used a mathematical expression for brilliance, called weighted light return or WLR, which we developed from the definition of *brilliance* given in the *GIA*

See end of article for About the Authors and Acknowledgments.
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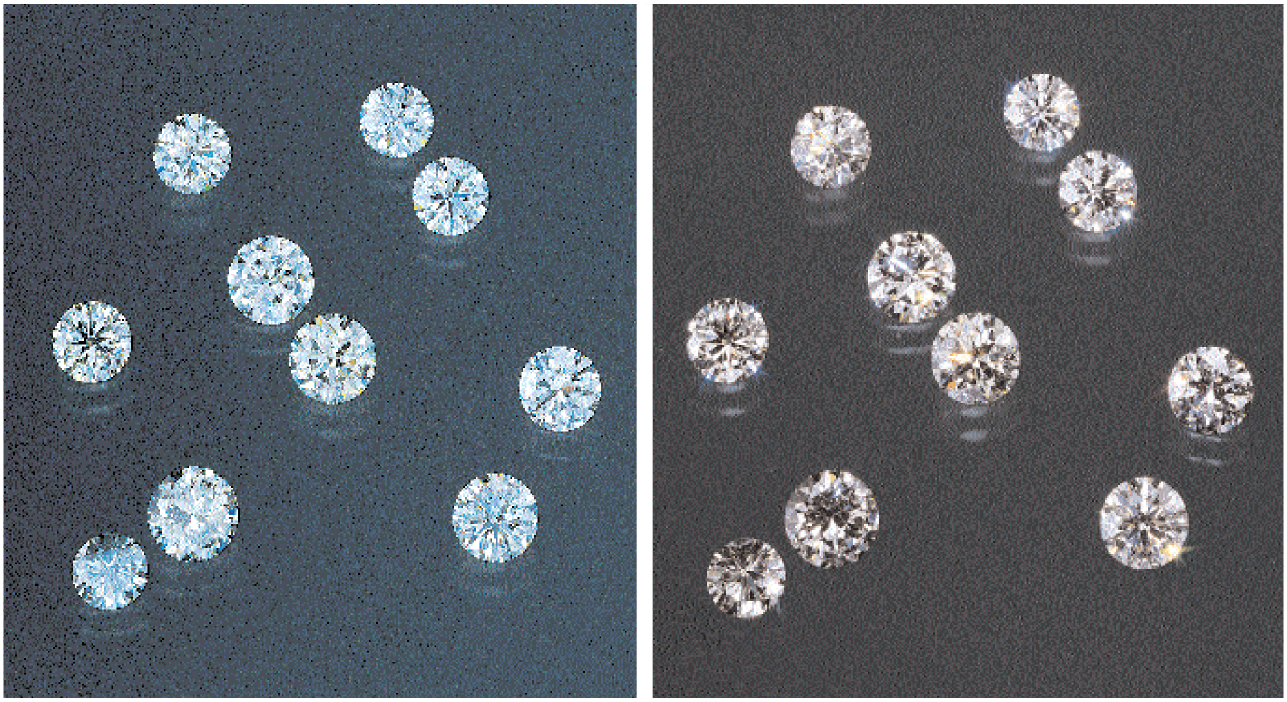


Figure 1. These 10 diamonds were selected to show a range of values for brilliance (as weighted light return, WLR) and fire (as dispersed colored light return, DCLR). They are, clockwise from the top right: RD13, 1, 6, 10, 25, 27, 24, and 23, with RD19 above RD21 in the center. Diffuse lighting, used for the photo on the left, emphasizes brilliance, represented by WLR; spot lighting, used to produce the image on the right, emphasizes fire, modeled as DCLR. The diamonds have been positioned so that WLR increases from left to right, and DCLR from bottom to top. For proportions and other information about these diamonds, see table 2. Photos by Harold & Erica Van Pelt.

Diamond Dictionary (1993)—that is, the intensity of the internal and external reflections of white light from the crown. (Note, however, that WLR does not include external reflections, i.e., glare.) The present installment of our research on the appearance of round brilliant diamonds addresses the effects of various proportions on fire. However, in the same dictionary, the definition for *fire* states merely "see Dispersion." Since all diamonds have the same dispersion value (0.044), this definition is not adequate. Rather, fire is the result of dispersion. Thus, we suggest a more direct definition: *fire is the visible extent of light dispersed into spectral colors*. As with WLR to express brilliance, we have developed a metric, or number, to express how well a round brilliant can disperse—or spread—light into colors (dispersed colored light return, or DCLR, as defined on p. 181 below).

In the present article, we discuss how dispersion creates the appearance of fire in a polished round brilliant diamond, present our metric for fire, and describe how this metric varies with changes in the proportion parameters. We also extend our earlier analysis of brilliance (WLR) over variations in two additional proportion parameters: star- and lower-girdle-facet lengths. Last, we compare the results from our exploration of fire to those from our earlier analysis of brilliance. We plan to address practical applications of our research to date in our next article, and to report on scintillation (the flashes of light reflected from the crown) in the future.

BACKGROUND

In Hemphill et al. (1998), we introduced our computer model for tracing light rays through a "virtual" diamond—a mathematical representation of a standard, 58-facet, round brilliant cut with a fully faceted girdle. The virtual diamond has perfect symmetry, so its exact shape can be described with eight parameters: crown angle, pavilion angle, table size, star facet length, lower-girdle facet length, culet size, girdle thickness, and number of girdle facets (figure 2). We scaled the values of most parameters (e.g., table percentage) to the diameter at the girdle, so the virtual diamond model applies to diamonds of any size. In addition, the virtual diamond has no inclusions, is perfectly polished, and is completely colorless. The model results include the power, position, exit angle, and color (i.e., wavelength) of all traced light rays; these results can be expressed both numerically and graphically.

BOX A: CURRENT PROPORTION GRADING SYSTEMS AND OTHER EVALUATIONS OF CUT

Trade debate on cut issues moved from the theoretical to the practical in the 1990s. The American Gem Society (AGS) opened a diamond grading laboratory in 1996 ("New AGS lab…," 1996), offering a cut grade for round brilliants that was modified from the conclusions set forth by Marcel Tolkowsky in *Diamond Design* (1919), as described and defined in the AGS Manual (American Gem Society, 1975). This system compares the proportions of the diamond to fixed ranges of crown angle, pavilion depth percentage, table percentage, girdle thickness, and culet size; it then assigns grades from 0 (best) to 10 (worst), for diamonds with good (or better) symmetry and finish. Several other organizations (for example, the Accredited Gemologists Association [AGA]; the Association of Gemological Laboratories, Japan [AGJ]; the International Gemological Institute [IGI]; and the Hoge Raad voor Diamant [HRD]) have their own grading systems that use different ranges of these proportions, as well as values of total depth percentage, to evaluate cut (Federman, 1997; Attrino, 1999). In each of these systems, the final grade is determined on the basis of the individual proportions, which are considered independently of one another. In addition, only small ranges of these individual proportions are assigned the highest grade, and deviations from these small ranges receive lower grades in each of these systems.

While the above laboratories use proportion measurements to evaluate diamond cut, others in the industry have used different approaches. Diamond Profile Laboratory pioneered a report with three types of photographic images that display cut information regarding the symmetry, dispersion, and light leakage of a polished diamond, independent of its proportions (Gilbertson, 1998). GemEx Systems produces a cut analysis report based on measurements taken with an imaging spectrophotometer using five light source positions (Roskin, 1999).

As branding has become more widely used for round brilliant cut diamonds, some manufacturing and retail firms have placed great emphasis on the proportions to which their diamonds are cut, while others have stressed the concept of light performance (see, e.g., "Hearts on Fire debuts…," 1997; "Perfectly cut…," 1997; Weldon, 1998). In addition, consumer- and trade-oriented Internet sites have hosted free-wheeling discussions about the many aspects of cut and its relationship to the appearance of a polished diamond (see, e.g., "Diamonds discussion forum," 2001]). Despite all this interest and effort, substantial differences of opinion continue to be expressed with regard to (1) whether there is a single set of proportions that produces the best appearance in a round brilliant diamond, and (2) how the proportions of a diamond affect the different aspects of its appearance.

In our analysis of brilliance, we chose a diffuse hemisphere of theoretical daylight (D65; see, e.g., Commission Internationale de L'Éclairage [C.I.E.], 1963) to illuminate this virtual diamond, and a hemisphere located at infinity for our observing surface, with a weighting function (cosine squared) that counted light rays that exit vertically more heavily than those that exit at shallow angles. We introduced the WLR metric for brilliance, and analyzed the values of this metric for about 20,000 combinations of crown angle, pavilion angle, and table size (see, e.g., table 1). The results showed that WLR depends on the *combination* of these three proportion parameters, rather than on the value of any one of them. They also showed that many diverse combinations of proportions produced similar WLR values.

We stated in 1998 that those results constituted only one part of the appearance of a round brilliant diamond, and that our virtual diamond would continue to be useful for exploring other appearance aspects. Within a computer model, we can control—and vary—the lighting and observing conditions, as well as work with large numbers of exact proportions that would be prohibitively expensive (or perhaps impossible) to manufacture as real diamonds. In addition, three important physical aspects of light interaction with a round brilliant cut diamond—three-dimensionality (3D), dispersivity, and polarization—can be readily incorporated into a computer model, although these aspects were omitted from earlier analyses of cut (see, e.g., Tolkowsky, 1919; Eulitz, 1974). More details on these physical aspects in diamond and other transparent materials can be found in Newton ([1730] 1959), Phillips (1971), Ditchburn (1976), and Born and Wolf (1980). A brief summary of the application of these aspects to diamond in particular can be found on the Internet (*GIA on diamond cut…*, 2001).

In short, diamond is a dispersive material: Its refractive index (R.I.) varies for different wavelengths (colors) of light. The dispersion value, 0.044, is the difference between diamond's R.I. for blue light (431 nm) and that for red light (687 nm). When a beam of white light enters a diamond at any angle other than perpendicular to the surface, it refracts, and the differences in R.I. among all the different

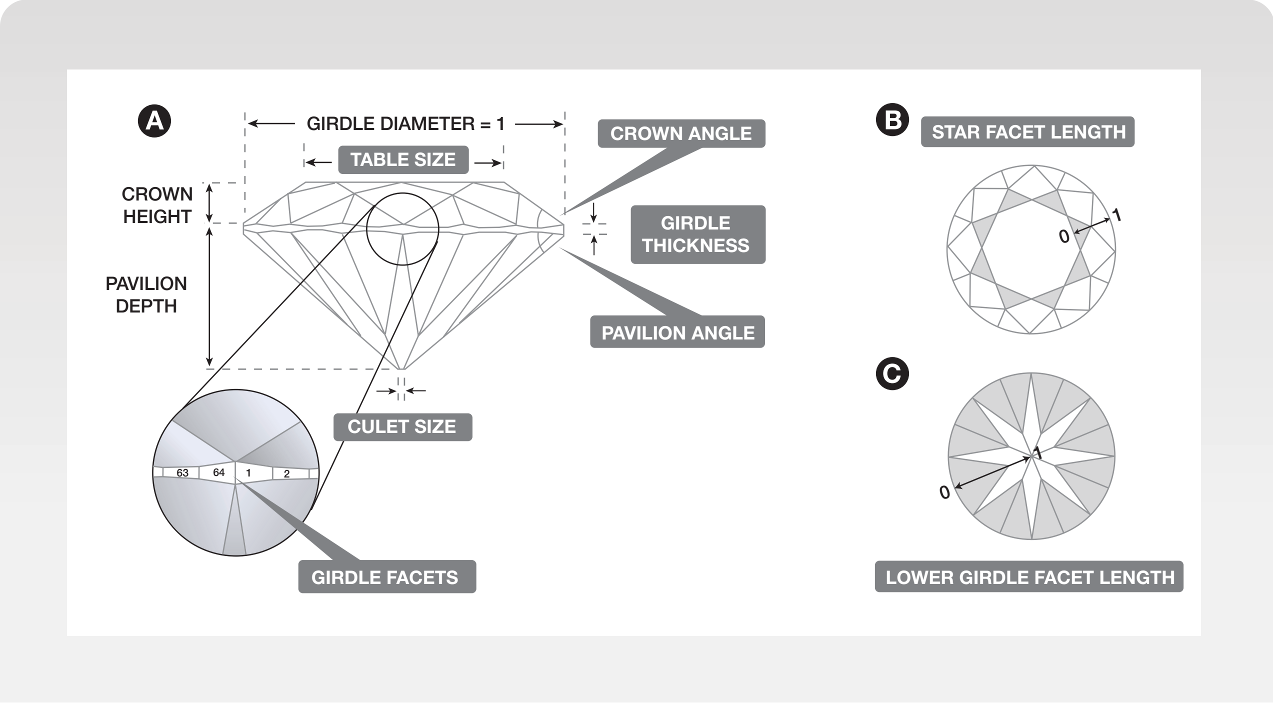


Figure 2. We used eight parameters—varied across the range given in table 1—to define our geometric model of the round brilliant shape. (A) All linear distances in this profile view can be described as a percentage of the girdle diameter. The enlarged view of the girdle is centered on the position where we measured the girdle thickness. (B) In this face-up view of the crown, the star facet length is shown at 50% so that the star facets extend half the distance from the table to the girdle (indicated here by 0–1). (C) In this face-down view of the pavilion, the lower-girdle facet length is shown at 75%, so that the lower-girdle facets extend three-fourths of the distance from the girdle to the culet (0–1). Adapted from Hemphill et al. (1998).

wavelengths cause the light to disperse, or spread, into its component colors. Although the initial spread of colors is less than 0.5°, these light rays of different wavelengths spread out further each time the light interacts with a facet; consequently, light rays of different colors that enter the diamond at the same spot, with the same orientation, can take totally different paths inside the diamond. The fire

TABLE 1. The eight proportion parameters that define the virtual diamond's shape, our reference proportions, and values used for calculations of DCLR and WLR.

Parameters	Reference proportions[a]	Values for DCLR	Increment for DCLR	Previous values for WLR	Values for new WLR results	Increment for WLR
Crown angle	34°	10°–46°	2°	19°–50°	20°–40°[b]	1°
Pavilion angle	40.5°	36°–45°	0.5°	38°–43°	38°–43°[b]	0.25°
Table size	56%	54%–68%	2%	50%–75%	53%–65%[b]	1%
Star facet length	50%	30%–74%; 50%–70%[b]	2%; 10%	5%–95%	30%–74%[b]	2%
Lower-girdle facet length	75%	45%–95%; 50%, 75%, 85%[b]	5%; N/A	50%–95%	50%, 75%, 85%[b]	N/A
Girdle thickness	3%	1.8%–4.4%	0.2%	1.8%–4%	Not varied	Not varied
Culet size	0.5%	0%[c]–20%	1%	0%[c]–20%	Not varied	Not varied
Number of girdle facets	64	Not varied	Not varied	16–144	Not varied	Not varied

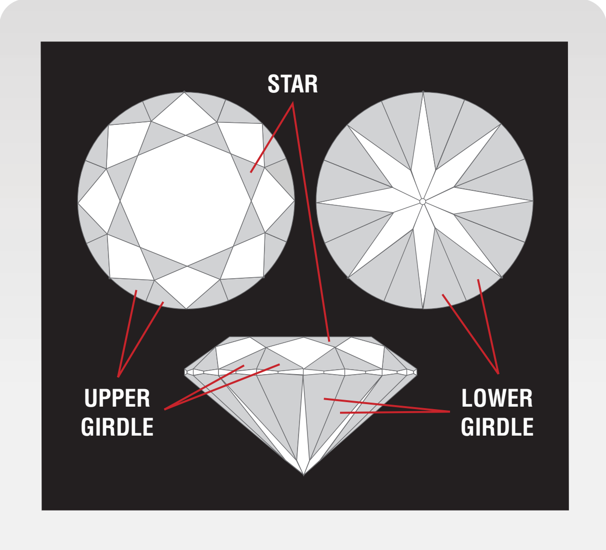
[a] *The reference value of a proportion is the default value chosen for that proportion in the cases for which it is not varied in the computer calculations.*
[b] *These proportions were used for the five-parameter simultaneous variation.*
[c] *A culet of 0% is also called no culet (or a pointed culet or pointed pavilion).*

one observes in a polished diamond is the net result of this dispersion of light.

In addition, because a round brilliant is made up of flat facets, light becomes partially polarized when it enters the diamond, and the polarization state then changes as the light moves within the diamond and interacts with several facets. This is important because the polarization state of a ray of light governs how much of its energy is reflected at various angles of incidence, for both internal and external reflections. The fraction of light that refracts and the fraction that reflects internally can be calculated accurately only by keeping track of the light's polarization state.

Three-dimensional light movement makes one aspect of diamond faceting obvious: For most commercially available proportions, star facets and upper- and lower-girdle facets together cover more than half the surface area of the round brilliant (figure 3). Although most cut analyses to date have focused on crown angle, pavilion angle, and table size, we cannot ignore the role that the star, upper-girdle, and lower-girdle facets play in the appearance of a typical round brilliant (our model does not require a separate parameter for upper-girdle facet length, because this parameter is determined by the combination of star length, crown angle, and table size). If the star length is approximately 50%, the star and upper-girdle facets make up about 40% of the surface area of the crown. Similarly, if the lower-girdle length is approximately 80%, the lower-girdle facets cover about 80% of the pavilion surface. The amount of light that refracts through and reflects from these facets is likely to be significant, but the two-dimensional analyses of cut found in the literature do not account for their contributions to appearance.

Figure 3. For most commercially available proportions, more than half the total surface area of the round brilliant is covered by star facets and upper- and lower-girdle facets. Because these facets also interact with light rays that enter the diamond and reflect within it, they must be considered in modeling the brilliance and fire of a diamond. However, most existing cut grading systems do not include evaluation of these facets.



Four teams of investigators (including the present authors) have modeled three-dimensional light movement in round brilliant cut diamonds, using a wavelength-dependent refractive index. The other three are: J. S. Dodson (1978, 1979); P. Shannon and S. Wilson, of Diamond Technologies Inc. (Shor, 1998; Shannon and Wilson, 1999); and a group at the Gemological Center in Lomonosov Moscow State University (abbreviated here as MSU) and OctoNus Software Ltd., headed by Y. Shelementiev and S. Sivovolenko ("Diamond cut study," 2001). Dodson kept track of the polarization component of each light ray so that its exact intensity could be calculated. The published literature does not indicate whether Shannon and Wilson included polarization in their model. The MSU group uses a "fixed polarization" approach, in which half the light energy is calculated in one polarization state and the other half is calculated in the state that is polarized perpendicular to the first. There are also some differences from one model to the next regarding the extent to which light rays are followed. The MSU group follows rays until they have interacted with at most 20 virtual facets, discounting any remaining energy a ray might have after this point. Our model follows all rays until at least 99% of their energy has been accounted for; some rays interact with more than 100 facets to reach this level.

Of course, real diamonds are subject to many more variations than are presently included in any of these models. For instance, inclusions and asymmetries may affect the appearance of a round brilliant as much as, or more than, variations in proportions. Color and fluorescence may also interact with a diamond's proportions to alter its appearance. In addition, it is well known among manufacturers that poor polish has a detrimental effect on diamond appearance regardless of the proportions. Last, grease and dirt on a diamond significantly degrade its appearance.

MATERIALS AND METHODS

Computation. For this work, we began with the same proprietary computer programs we used previously, and wrote several more computer routines to represent additional light sources and to calculate results relevant to the dispersion of light. As we did in 1998, we verified these new programs by running a test program for which the solution was found manually. Our programs run on any computer that accepts programs written in C; we used 16 Pentium III and four Pentium II processors in conventional desktop computers to carry out the calculations presented here. With these programs, we calculated the fire metric (DCLR) for more than 26,000 round brilliant proportion combinations (again, see table 1). On average, these calculations took 1.5–2 processor hours each.

Starting with 5,733 typical combinations of crown angle, pavilion angle, and table size (from the ranges used in Hemphill et al., 1998), we also calculated WLR values with various star and lower-girdle facet lengths, keeping girdle thickness, culet size, and the number of girdle facets fixed at our reference proportions (again, see table 1). This resulted in 395,577 additional WLR values, using 23 star facet lengths (from 30%–74%, in increments of 2%), and three lower-girdle facet lengths (short—50%, medium—75%, and long—85%). The medium and long lower facet lengths were chosen as typical values seen in the trade today, while the short (50%) value was chosen because this is the lower-girdle length Tolkowsky (1919) used (see also Green et al., 2001).

Diamonds. We obtained (or had manufactured) 28 round brilliant diamonds (0.44–0.89 ct), some with unconventional proportions (see table 2). The 45 "view from infinity" (VFI) diagrams described below

TABLE 2. Proportions and calculated metric values for 28 diamonds examined for this study.[a]

Sample no.	Weight (ct)	Clarity grade	Color grade	Crown angle (°)	Pavilion angle (°)	Table size (%)	Star facet length (%)	Lower-girdle facet length (%)	Girdle thickness (%)	Culet size (%)	WLR	DCLR
RD01	0.61	VS$_1$	E	34.3	40.6	54	53.8	81	2.9	0.91	0.283	3.97
RD02	0.64	SI$_2$	E	32.9	41.5	59	54.7	77	4.5	1.09	0.277	3.41
RD03	0.55	VS$_2$	H	32.0	40.9	63	60.3	80	3.7	0.75	0.272	3.39
RD04	0.70	VVS$_2$	E	36.2	41.9	58	57.7	79	5.6	0.73	0.261	3.10
RD05	0.66	VS$_2$	F	24.1	42.2	58	56.5	83	3.6	0.69	0.294	2.86
RD06	0.59	VVS$_2$	F	23.1	41.9	57	60.6	78	3.2	1.07	0.301	2.87
RD07	0.76	SI$_1$	F	36.4	41.5	53	59.4	89	3.1	1.72	0.271	3.46
RD08	0.50	VVS$_1$	H	33.4	41.2	58	54.0	84	3.9	0.97	0.279	3.79
RD09	0.66	IF	F	23.6	42.1	56	58.8	80	4.5	1.04	0.300	2.92
RD10	0.68	VS$_2$	G	34.9	40.9	54	54.7	76	3.0	0.70	0.281	3.89
RD11	0.71	VS$_2$	D	37.2	41.9	58	49.1	87	4.3	0.89	0.262	3.21
RD12	0.71	SI$_1$	F	35.0	41.0	57	58.5	76	4.6	0.71	0.274	3.52
RD13	0.59	VVS$_2$	E	33.7	41.1	52	63.0	80	3.3	1.11	0.281	4.01
RD14	0.71	SI$_1$	G	34.5	42.1	59	60.9	80	3.5	1.05	0.276	2.87
RD15	0.67	VS$_1$	H	25.7	40.6	59	54.2	76	3.4	0.68	0.291	3.37
RD16	0.82	VS$_1$	G	33.8	40.4	54	51.9	76	3.3	0.82	0.281	3.77
RD17	0.75	VS$_2$	F	26.0	38.4	60	51.3	75	3.5	0.97	0.283	3.08
RD18	0.62	VVS$_2$	H	29.1	41.2	61	46.9	75	3.3	0.88	0.281	3.16
RD19	0.72	VS$_1$	H	29.2	39.5	63	51.7	76	3.3	0.83	0.276	3.41
RD20	0.62	VVS$_1$	I	34.3	40.7	61	55.1	79	3.2	1.26	0.279	3.63
RD21	0.82	VVS$_1$	I	35.8	41.2	57	57.3	76	3.7	0.83	0.275	3.14
RD22	0.81	VS$_1$	K	35.9	39.2	55	54.2	77	3.3	0.83	0.274	3.75
RD23	0.72	VVS$_2$	I	36.6	40.5	54	55.9	79	4.0	1.23	0.269	3.92
RD24	0.58	VVS$_1$	H	35.8	38.8	66	58.9	79	4.0	0.91	0.259	3.23
RD25	0.82	VVS$_2$	H	39.9	41.8	70	53.2	76	3.0	0.82	0.253	2.18
RD26	0.89	VS$_1$	I	38.0	42.0	61	56.9	74	3.6	0.98	0.261	2.66
RD27	0.44	VS$_2$	G	11.1	50.7	63	51.8	77	3.4	1.00	0.213	1.06
RD29[b]	0.69	SI$_1$	F	37.7	41.9	61	50.5	76	3.0	1.07	0.267	2.76

[a] *All samples showed good or very good symmetry.*
[b] *Sample RD28 was not included in this study because it has a modified facet arrangement.*

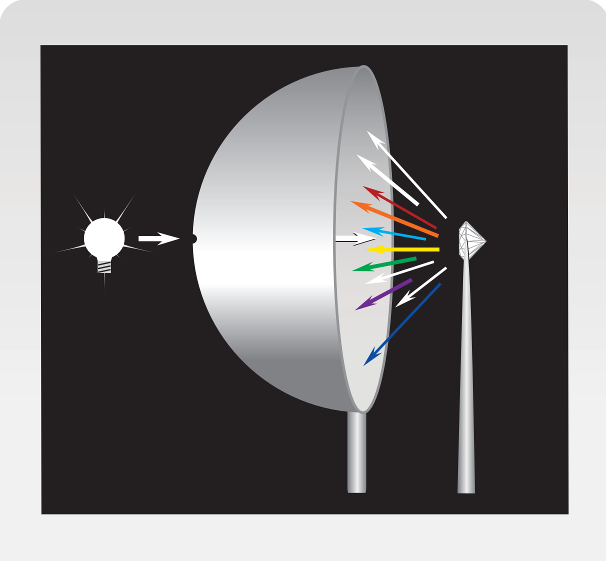


Figure 4. This experimental design allowed us to observe dispersed light from actual diamonds. A white hemisphere 40.6 cm (16 inches) in diameter is used as the observing surface, and a 0.95 cm hole at the center allows light from behind the hemisphere to shine on a diamond. The round brilliant is centered in the beam of light, with its girdle as close as feasible to the plane of the hemisphere's rim and its table oriented perpendicular to the beam. The light source is located about 20 cm behind the hole, so that light shining on the diamond is largely composed of parallel rays. Sizes and distances in this diagram are not to scale.

were calculated for these 28 diamonds and for another 17 diamonds (in the same weight range) with very good or excellent symmetry and polish that were chosen at random from those graded at the GIA Gem Trade Laboratory (GIA GTL). We examined and recorded dispersed patterns of colored light (see next section) for these diamonds and for more than 400 round brilliants chosen from the same GIA GTL population using the same symmetry and polish criteria.

Conditions for Observing Fire. To analyze brilliance, we chose the diffuse lighting condition we reported on in 1998, specifically because it maximized the effects of white light return while minimizing the impact of fire and contrast. However, because this lighting condition suppresses fire, it would not be appropriate for exploring the effect of proportion combinations on this appearance aspect. Again, fire in a round brilliant is the display of pure spectral hues that arise because the diamond is behaving like a prism, dispersing white light into its color components. Fire is not seen as a uniform color across the entire crown of the diamond at once, or as a single rainbow, but as localized flashes of various colors that change depending on the position from which the diamond is viewed.

Keeping these aspects in mind, we began our search for a suitable lighting condition with the observation that diamonds look fiery under both sunlight and spot lights (such as the halogen lights found in many retail stores). This lighting is *directed*, that is, it comes from a very small area (relatively speaking), in contrast to diffuse lighting, which comes from all directions (such as outdoors on a foggy day, or fluorescent lighting reflected off white ceilings). These impressions were further supported by our experience photographing polished diamonds to capture their displays of fire. With either film or a digital camera, we found that a source of directed light was needed (in combination with diffused tungsten photography lights) to see fire in a photographic image (again, see figure 1).

Tolkowsky (1919) suggested a way to view fire from a diamond, that is, by using a sheet of paper with a pinhole. Light shining through the hole in the paper falls on the table of the diamond, and dispersed light that returns through the crown can be observed on the side of the paper facing the diamond. In such an illumination geometry, however, angular relationships are distorted, and the distance between the diamond and the paper strongly affects how much of the light returning through the crown is visible.

Using the same general idea, we created an apparatus to view and capture images of dispersed light from actual diamonds. As shown in figure 4, we introduced light through a 0.95 cm hole in a white plastic hemisphere (40.6 cm [16 inches] in diameter), which was the observing surface. We used a Lumina fiber-optic light, model Fo-150, which has a color temperature of 2920 K, and placed it approximately 20 cm from the hole, so that a preponderance of the light rays falling on the diamond were parallel to one another. The beam of light was centered on the diamond's table, and perpendicular to it. The diameter of this hemisphere was about 70 times larger than that of the diamonds we examined. The light emerging from the diamond could be viewed on the hemisphere, or recorded as a photograph, such as the one shown in figure 5.

We call the colored patches on the surface of the hemisphere *chromatic flares*. (In contrast, *fire* is seen when we observe the diamond directly.) Observation of chromatic flares requires an edge to the light source, specifically, a strong difference between light and dark: The dark portion contributes no color, so dispersion is emphasized.

Many of the flares are quite small, but some are wide, or long, or both (again, see figure 5). Any particular chromatic flare may include the whole spectrum of colors or only a few of them. Furthermore, if the dome is increased in size, most flares may spread out. The overall light level in the room strongly affects how many chromatic flares an observer sees; we saw many more flares in the dome in a darkened room than with the room lights on.

We used a Minolta X700 camera with a 45-mm lens, ASA 100 film, and an f-stop of 18 to take photographs of light dispersed from real diamonds. Exposure times varied from 45 to 150 seconds. We examined and recorded dispersed patterns of colored light for the more than 400 round brilliants mentioned earlier, to gain an understanding of the range of appearances of chromatic flares, and thus of fire in polished diamonds.

Model Conditions for Fire. We chose to model directed lighting as a bright point source of D65 illumination (a common model for average daylight; see again C.I.E., 1963), located very far from the diamond (at infinite distance) and centered over the diamond's table. With this condition, the unpolarized light rays entering the crown facets are parallel to one another and perpendicular to the table (figure 6, left). The entire crown is illuminated.

An observer position also needed to be determined. As the distance between the observer and the diamond increases, the observer sees less white light and more dispersed colors, but he or she can see only some of the colors at one time (figure 6, right). Conversely, when a round brilliant is viewed close up, the fire is less discernable since various colors viewed close together appear as white light. (This effect also was demonstrated in our apparatus for observing chromatic flares from actual diamonds, where some of the dispersed light output from the diamonds appeared as white light.) Therefore, to observe maximum fire, the observer must be as far away from the diamond as possible. In addition, the assessment of fire requires multiple views of the diamond, from different viewing angles. (The fact that these multiple views cannot be made simultaneously, but must be made sequentially from various positions, may complicate the assessment of fire in a diamond by an "actual" observer, but can be incorporated relatively easily in a model.) Our "observer" for WLR was a hemisphere at infinity (with a cosine squared weighting function for the returned light), and we found that this observer was a good choice for viewing fire as well. This model observer "views" the virtual diamond from all angles, while the weighting factor incorporates the importance of the face-up position. This combination of lighting and observing conditions tests the *maximum extent* to which a round brilliant with a particular choice of proportions can disperse light into its component colors.

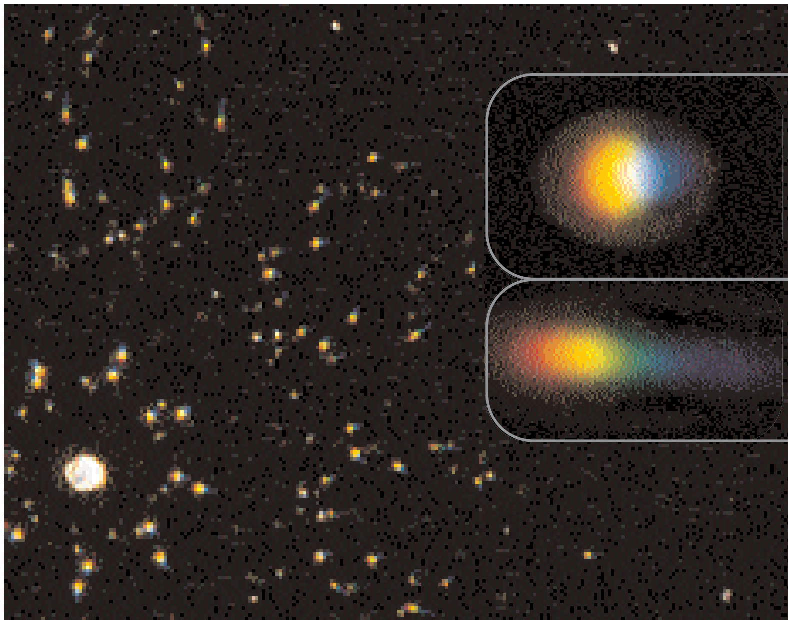


Figure 5. This image shows a variety of chromatic flares (colored light patterns) that were produced by a round brilliant diamond on our observation hemisphere. Larger images of two chromatic flares, shown as insets, illustrate some of the variety seen in these patterns. Photo by Al Gilbertson.

A Metric for Fire: DCLR. To analyze the fire from a virtual diamond graphically, we plotted the model output of our observing hemisphere, projected onto a flat plane, using a polar projection in which the distance of any plotted point from the center of the diagram is proportional to the exit angle of that ray. We call this graphic result a *view from infinity* (VFI) diagram. The combination of point light source and infinite viewing distance yields only dispersed light on the observing hemisphere; that is, the result appears as various colored streaks (with no white centers). These streaks are composed of colored spots showing the final exit directions of individual rays. The *colored streaks* on a VFI diagram for a virtual diamond correspond to the chromatic flares on the hemisphere for an actual diamond of the same proportions.

Figure 7 shows a complete VFI diagram for a virtual round brilliant cut diamond of our reference proportions. For a diamond with perfect symmetry (such as our virtual diamond), all of this information

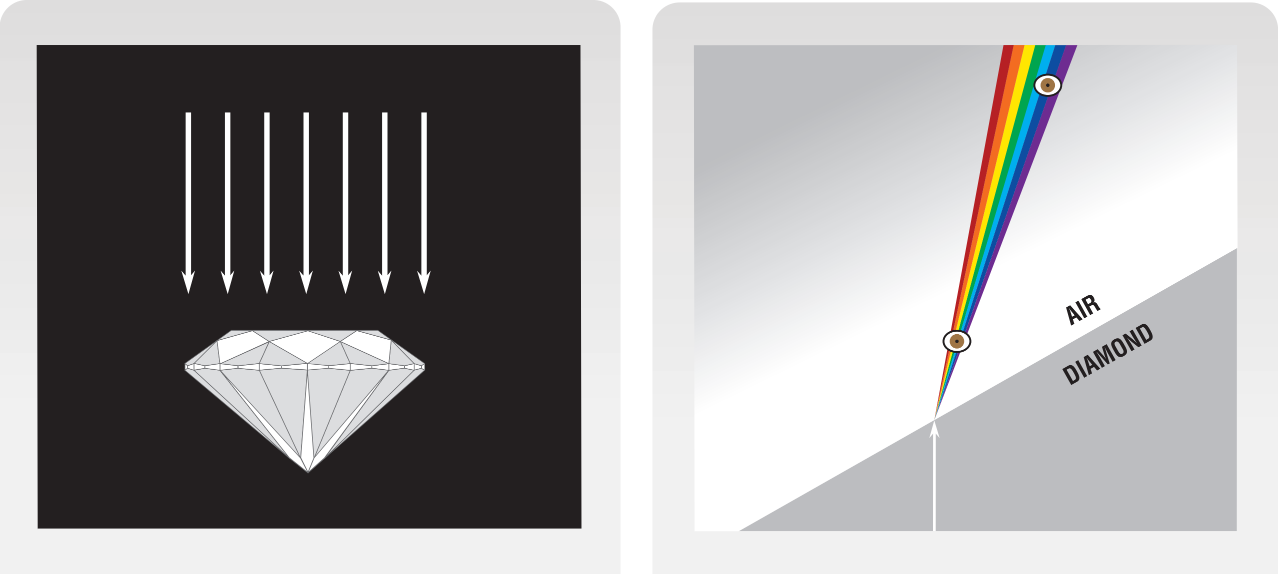
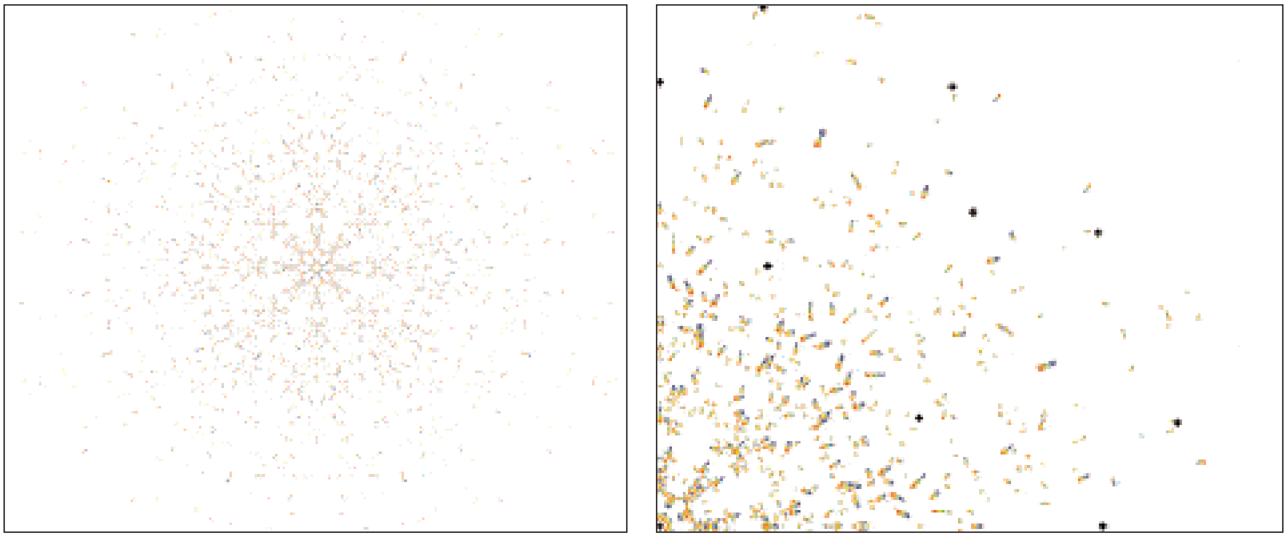


Figure 6. Left: In our model lighting condition for fire, a point source located at infinity produces parallel rays entering the crown of the diamond, perpendicular to the table. Right: As dispersing light moves away from its source (here, the surface of a diamond), the different wavelengths spread out in space. When a small detector, such as a human eye, is far enough away (shown as the upper oval), only some of the colors can be seen from any one viewpoint, so movement of the diamond (or observer) is necessary to see the other colors. When the detector (lower oval) is close enough to the surface of the diamond, all the colors are present, so it "sees" predominantly white light.

is in fact contained in a one-eighth "pie slice" of the diagram. The angle at which rays exit from the crown is shown by their position; rays that exit perpendicular to the table are displayed in the center of the diagram, while those that exit close to the horizontal appear around the circumference of the circle. We can take advantage of the symmetry and plot only one-fourth of this diagram on a sheet of paper,

Figure 7. This "view from infinity" (VFI) diagram is calculated for a virtual round brilliant cut diamond of our reference proportions (34° crown angle, 40.5° pavilion angle, 56% table, 50% star facet length, 75% lower-girdle facet length, thin to medium girdle, 0.5% culet, and 64 girdle facets). Rays of dispersed light that emerge straight up from the round brilliant are displayed in the center of the diagram, whereas rays that emerge close to the horizontal are shown near the edge of the circle. The perfect eight-fold symmetry of the virtual diamond allows us to portray all the information for the diamond using a small part of such a diagram. For example, on the right, a quarter of the diagram is shown with each ray's line width proportional to its brightness.



with the different intensities of the rays appearing as different plotted thicknesses (figure 7, right).

We calculated VFI diagrams for proportion combinations taken from 45 actual diamonds, to compare with their observed dispersion patterns. (The detailed measurements of each round brilliant were averaged to produce a symmetrical proportion combination for that diamond.) Figure 8 shows three actual diamonds with different proportions as well as the VFI diagrams calculated for them. Although these color-streak patterns look different, we had no way to evaluate these VFI images quantitatively: the diamonds appear to show comparable fire. We needed a numerical value—that is, a metric—that could be used to evaluate fire for thousands of proportion combinations. It was important that the metric incorporate factors that matter to people when they observe fire in round brilliants; it also had to produce numerical values that differentiate a very fiery diamond from one with little fire.

The VFI diagrams display a variety of properties that can be combined into a metric, such as the total number and relative brightness of colored spots, and the lengths and angular distribution of the colored streaks made up of these spots. The metric we derived—*dispersed colored light return*, or *DCLR*—describes the potential of a round brilliant diamond with given proportions to display dispersed light when viewed face-up. Mathematically, DCLR is defined as:

$$\mathrm{DCLR} = \Sigma_{\mathrm{streaks}}\, \Sigma_{\mathrm{colors}}\, (\mathrm{Area} \times \mathrm{Smoothed\ Intensity} \times \mathrm{Weighting\ Factor})$$

That is, DCLR is the sum over all colored streaks, of the sum over all colors (sampled every 10 nm; see again C.I.E., 1963), of the size (area) of each colored streak multiplied times the "smoothed"* brightness (intensity) of each spot along the streak, times an exit-angle weighting factor (the square of the cosine of the ray's exit angle, which we also used for WLR).

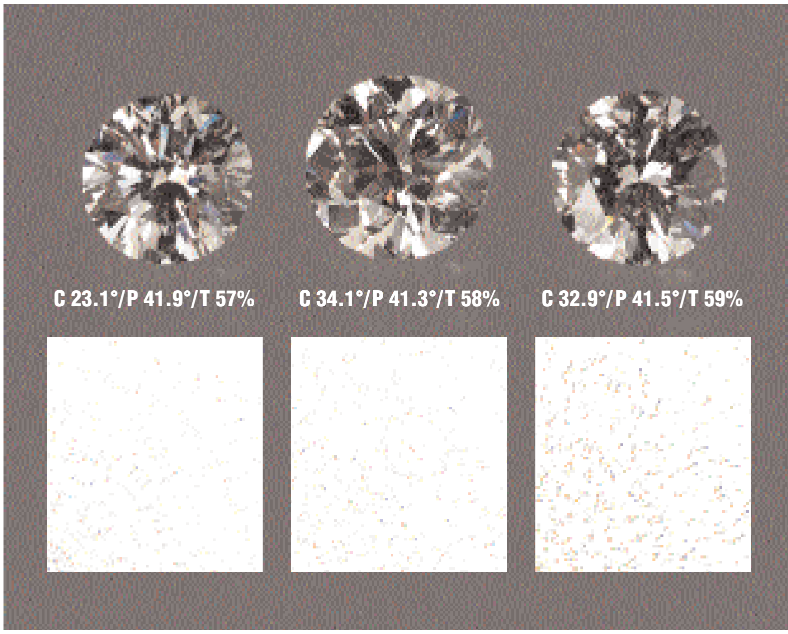


Figure 8. These three diamonds (0.50–0.64 ct) have rather different proportions (crown, pavilion, and table given here), and the VFI diagrams calculated for their proportions show different patterns. However, all three diamonds appear bright and display comparable fire. Photo by Elizabeth Schrader.

The VFI diagrams show additional properties that we chose not to include in DCLR. For example, we included the angular distribution of the colored streaks in this metric, but not a radial term (which would have, e.g., differentiated flares coming from bezel facets from flares coming from star facets), because we believe that a human observer cares about fire from the diamond as a whole. Similarly, we did not use the color distribution within a given streak (that is, whether it contains a whole rainbow or only a few colors) or the orientation of a streak, choosing instead to focus on the overall impact of colored flashes. Last, we did not consider the color distribution of the spots (i.e., the relative number of red spots to green, yellow, or blue spots), because the various VFI diagrams we plotted showed balanced representations of all colors, a property we also observed in real diamonds.

Before starting the calculations, we needed to establish an appropriate brightness threshold for DCLR, to determine the range between the brightest and dimmest rays a person could be expected to see against a generally bright background (as in the light we typically use in our homes or offices, whether fluorescent or incandescent). The scientific literature dealing with human vision contains several works about the least amount of light that can be seen, or the brightest light in which objects can be discriminated, but almost nothing about the range of light levels perceived by humans in

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**We used a smoothing function because the intensity of calculated colored spots varied over a large range. Although we wanted brighter rays to count more than dimmer rays, we did not want the large scale of these numbers to overwhelm other factors, such as the area. Thus, rather than use the intensity directly, we "smoothed" it with an "S-shaped" function. The center of the "S" set the intensity of colored streaks to be included, based on their brightness relative to the strongest rays, and the smoothed function avoided an abrupt transition between the included rays and slightly dimmer excluded rays.*

BOX B: COMPARISON OF MODELED RESULTS TO ACTUAL ROUND BRILLIANT PROPORTIONS

We analyzed interpolated DCLR values for the proportions of 67,943 round brilliant cut diamonds sent to the GIA Gem Trade Laboratory for grading to discover the range and distribution of DCLR shown by a group of typical commercial diamonds. ("Interpolated" means that we estimated the value for each proportion set from the values of their nearest neighbors in proportion space among the 26,000 points we had calculated.) This population of diamonds had crown angles ranging from 19.6° to 45°, pavilion angles from 36.1° to 44.8°, and table sizes from 54% to 68%. Star facet lengths were assumed to be 50% and lower-girdle facet lengths were assumed to be 75%, chosen to be at the reference values (see table 1 in the text) because we lacked these proportion measurements for this group of diamonds. These proportion combinations yielded DCLR values from 1.5 to 4.3, with a mean of 3.1.

The 680 sets of proportions (1% of the total) that yielded the highest DCLR values (3.5 or greater) had crown angles ranging from 25.8° to 42.8°, with most falling in the narrower range of 31° to 37°; pavilion angles of 36.2° to 41.5°, with the majority falling between 39.0° and 41.0°; and table sizes of 54% to 68%, with most in the smaller range of 54% to 63%. In contrast, the 1% of proportion combinations that yielded the lowest DCLR values (2.2 or less) had crown angles from 26.9° to 40.9°, with most in the narrower range of 32.0° to 37.0°; steeper pavilion angles, from 41.8° to 44.8°, with most being greater than 42.0°; and tables that varied from 57% to 68%, with most 62% or larger.

Also relevant to the evaluation of modeled results for fire is the large population of proportion sets with DCLR values near the mean. We therefore examined the proportions of the middle 68% of the entire group; those proportions yielded DCLR values between 2.85 and 3.40. The range of each proportion for this group was the same as for the whole group of 67,943 diamonds. However, the majority of the crown angles for these diamonds fell in the narrower segment of 26.5° to 39.5°; only eight diamonds had crown angles of 43° or greater. Most pavilion angles were between 40.0° and 42.6°, and typical table sizes were 54% to 66%.

ordinary lighting situations (Dr. R. Brown, pers. comm., 2000). The only clear point of agreement we found is that this range is greater than the 256 tones (grayscale) used by a computer monitor (see, e.g., Begbie, 1969; Boynton, 1979).

Therefore, we empirically derived an estimate of the brightness of the least intense flare a person could be expected to perceive, using a combination of (1) the hemisphere described above for observing fire in actual diamonds, (2) four diamonds, and (3) four human observers. The observers compared the actual patterns of chromatic flares displayed by each diamond to six VFI diagrams calculated for that diamond's proportions at different brightness thresholds. The observers agreed strongly in all four cases, which eliminated the need for more extensive testing. This comparison revealed that human observers see chromatic flares over a brightness range of about 3,000 against a background of ordinary room light: That is, the dimmest rays seen were 3,000 times less bright than the brightest rays. Consequently, we set the threshold at that level (3.5 orders of magnitude) for the calculations presented here.

The diamonds in figure 1 span DCLR values from 1.1 to 4.0, and show a range of appearances of fire. Under standardized lighting conditions (with sufficient directed light), we found that trained observers may see differences in DCLR levels of 0.5, and readily see differences of 1.0. We interpolated DCLR values for the averaged proportions of 67,943 round brilliants received in the GIA Gem Trade Laboratory for diamond grading, to evaluate the numerical range and distribution of DCLR in a commercially relevant group of real diamonds (see box B). For the purposes of this article only, and for the convenience of the reader, we offer three categories for DCLR, based on the distribution of DCLR values for the diamonds in box B:

Above average	> 3.5
Average	2.8–3.5
Below average	< 2.8

These categories should not be taken as fire "grades." They are offered as a convenience only, to compare the relative display of fire for virtual diamonds of various proportions.

RESULTS

VFI Plots. We determined that there was a good match between the chromatic flares we saw from the 45 actual samples and the VFI diagrams calculated for their proportions. Figure 9 compares the photograph of the chromatic flares from a 0.61 ct round brilliant with high symmetry to a VFI diagram for a virtual diamond with the averaged proportions of this actual diamond. The positions of the calculated colored streaks are an excellent match for the positions of chromatic flares recorded from the diamond. However, the display of colors in the chromatic flares is compressed (i.e., colors recombine into white light), both because the photographed hemisphere is much closer to the actual diamond than the modeled hemisphere is to the virtual diamond and because of the limitations of photography. All 45 calculated VFI diagrams showed similar matches to the corresponding chromatic flare photographs, despite the differences between the virtual and actual diamonds (e.g., asymmetries, inclusions, color, or fluorescence).

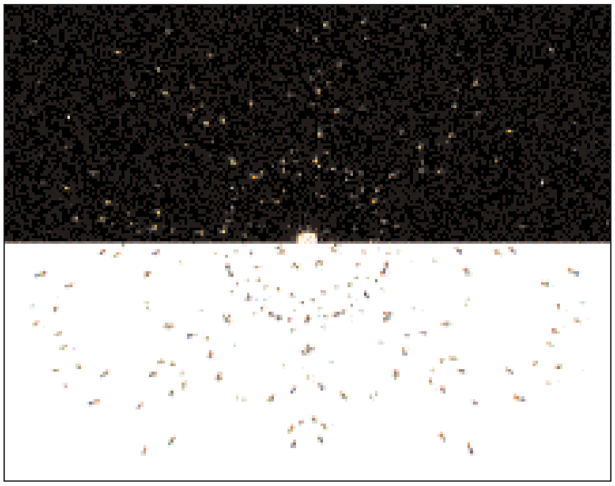


Figure 9. Comparison of the pattern of chromatic flares from a 0.61 ct round brilliant with very high symmetry (top, photographed on the hemisphere described in figure 4) to the VFI diagram calculated for a virtual diamond with the averaged proportions of the actual diamond (bottom) reveals that the calculated diagram matches the positions of real light output quite well. Photo (top) by Al Gilbertson.

Dependence of DCLR on Proportions. As of July 2001, we had calculated DCLR values for more than 26,000 proportion combinations, varying seven of the eight model parameters independently and five of the eight model parameters simultaneously (again, see table 1). We found that DCLR depends on these parameters singly and in combination. In other words, DCLR, like WLR, can be maximized by proportion combinations in a number of different ways.

Results for Individual Parameters. Our investigation of the dependence of DCLR on proportions began with an examination of how DCLR varies with each parameter while the remaining seven parameters are held constant. Except where otherwise noted, we fixed the remaining parameters at the reference proportions given in table 1.

Crown Angle. In general, DCLR increases as crown angle increases; but, as figure 10A shows, DCLR hovers around a value of 3.5 for crown angles between 20° and 40°. There are two local maxima in this region, at about 25° and 34°, and DCLR rises steeply for crown angles greater than 41°. Note that moderately high crown angles of 36°–40° yield a slightly lower DCLR value than either of the local maxima.

Pavilion Angle. Diamond manufacturers often cite this parameter as the one that matters most for brilliance (e.g., G. Kaplan, pers. comm., 1998). In Hemphill et al. (1998), we reported that most "slices" of the WLR data that varied only pavilion angle showed a sharp maximum at one angle (see, e.g., figure 5 of that work), although which pavilion angle gave the highest WLR depended on the other parameters. We found a substantial, but uneven, decrease in DCLR for pavilion angles between 38° and 43°; this is the single parameter that caused the most significant variation across the commercially common range. Figure 10B shows an overall decrease in DCLR with increasing pavilion angle, with an overall maximum at 38.5°, and a local maximum ("hump") at 40°–41°.

Table Size. DCLR generally decreases as table size increases, with the values falling into three distinct regions: DCLR is higher for small tables (less than 58%), approximately constant for table sizes between 58% and 64%, and decreases further for larger tables, as shown in figure 10C.

Star Facet Length. We calculated the variation in DCLR with changes in the length of the star facet for three values of the crown angle: 25°, 34°, and 36° (figure 10D). (The angles of the star and upper-girdle facets relative to the girdle plane depend on both the star facet length and the crown angle. Thus, we expected DCLR results for different star lengths to vary at different crown angles.) Although the range of DCLR values is relatively small, each curve shows a clear maximum. At the reference crown angle of 34°, a star length of about 64%–65% yields the highest DCLR. This maximum shifts to about 56% for a crown angle of 36°. In our shallow (25°) crown example, the maximum DCLR value is found for 52%–54% star lengths.

Lower-Girdle Facet Length. One of the most dramatic results was the variation in DCLR with changes in lower-girdle length (figure 10E). DCLR values climbed from below average to above average as the length of the lower-girdle facets increased from 45% to 85%, but then fell as this parameter further increased to 95%. As the lower-girdle facets get longer, they make a shallower angle with the girdle plane (closer to the pavilion angle) and very slightly shallower angles with each other and with the pavilion mains on either side. They also cause the pavilion mains to be narrower.

Girdle Thickness and Culet Size. DCLR showed very little change over the whole range of girdle thickness, with a slight minimum for a medium girdle (figure 10F). DCLR drops fairly smoothly as culet size increases from none to extremely large (figure 10G).

Combined Effects. Ideally, we would like to have shown the combined effects of crown angle, pavilion angle, table size, and star and lower-girdle facet length on DCLR in one graphic image. However, only two of these independent proportion variables can be displayed with the complete variation of a dependent value (such as DCLR) on a single graph. One way to illustrate the effects of two parameters is to draw contours showing ranges of DCLR values, similar to the WLR contours in Hemphill et al. (1998, figures 7–11). The "peaks" on such a contour plot represent proportion combinations that produce the highest calculated DCLR values. By grouping several contour plots together, we can show the results when two additional proportions are varied.

The nine contour plots in figure 11 show DCLR values with variations in both crown angle and pavilion angle, for a table size of 60%. They also demonstrate the effects of varying star and lower-girdle facet length, for three values of each. This figure contains a large amount of information about how these proportions work together to change DCLR.

The graph in the bottom center of figure 11 contains the point closest to our reference proportions (i.e., 34° crown angle and 40.5° pavilion angle), marked "R", with a DCLR of 3.38. For this 60% table size, 50% star, and 75% lower-girdle facet length, the highest DCLR values are found at low pavilion angles of 36°–37° and a high crown angle of 46°. DCLR decreases sharply for most crown angles at high pavilion angles (greater than 42°). However, the DCLR at shallow crown angles depends strongly on the pavilion angle as well.

Within the proportion space shown on this bottom center plot, there are two "ridges" of higher DCLR. These ridges represent combinations of crown and pavilion angles that work together to produce higher DCLR. One ridge ends at about a 16° crown angle and a 43° pavilion angle; DCLR decreases from there for both shallower and steeper pavilion angles. The other ridge, to the right, is broader and less distinct in this specific plot.

As we compare the topography shown on this plot with that of each of the other eight plots in figure 11, we can see: (1) the strong effect of lower-girdle facet length on DCLR (compare plots left-to-right); and (2) the weaker, but still significant, effect of different star facet lengths (compare plots top-to-bottom). Shorter lower-girdle facets greatly decrease DCLR for most combinations of crown and pavilion angle, with a few exceptions (e.g., for crown angles greater than 40°, or for crown angles of 22°–46° with pavilion angles of less than 37°). Longer lower-girdle facets yield a large number of crown and pavilion angle combinations with average or above-average DCLR.

In a broad region (i.e., with crown angles from 16° to 46° and pavilion angles from 36° to about 43°), the combination of star and lower-girdle facet lengths changes the location of the two ridges of higher DCLR, and alters the depth of the valley between the ridges. Overall, the center plot (for 60% star and 75% lower-girdle facet length) shows the largest number of crown and pavilion angle combinations that yield average and above-average DCLR values, but the upper-right plot (for 70% star and 85% lower-girdle facet length) shows the most combinations that yield DCLR values of 4.0 or higher.

The fifth proportion we varied was table size. The three regions found for DCLR variations with table size alone (i.e., high for small tables, approximately constant for table sizes between 58% and 62%, and lower for larger tables in figure 10C) held true for the most part in the multi-dimensional exploration as well. Figure 12 shows three contour plots for 54% (small), 60% (medium), and 66% (large) table sizes, with 50% star and 75% lower-girdle facet lengths (the reference values). Many more combinations of crown and pavilion angles with small tables yielded average or greater DCLR values than those with large tables. Although there were many crown and pavilion angle combinations that yielded these DCLR values with a 60% table, on average DCLR values were lower than with a small table. For large tables, only a narrow range of pavilion angles (shallower than typical) produced these DCLR values.

The combined effects of table size and lower-girdle facet length are shown in figure 13 for several significant combinations of crown and pavilion angle. We found that longer lower-girdle facet lengths generally yield higher DCLR values than

Figure 10. DCLR changes as one parameter is varied and the others are held constant at reference proportions (table 1). Across commercial ranges, pavilion angle, lower-girdle facet length, and table size have the largest effect. (A) DCLR generally increases as crown angle increases, although it is nearly constant from 20° to 40°, with local maxima at 26° and 34°. (B) DCLR generally decreases as pavilion angle increases, with local maxima at 38.5° and 40.5°. (C) DCLR decreases gradually as table size increases, varying little from 58% to 64%. (D) Star facet length changes yield small, but quite variable, DCLR differences. (E) DCLR increases with lower-girdle facet length, up to 85%. (F) DCLR changes little with girdle thickness, but there is a small minimum at about 3.0%. (G) DCLR decreases as culet size increases, but does not go below average until the culet is extremely large.

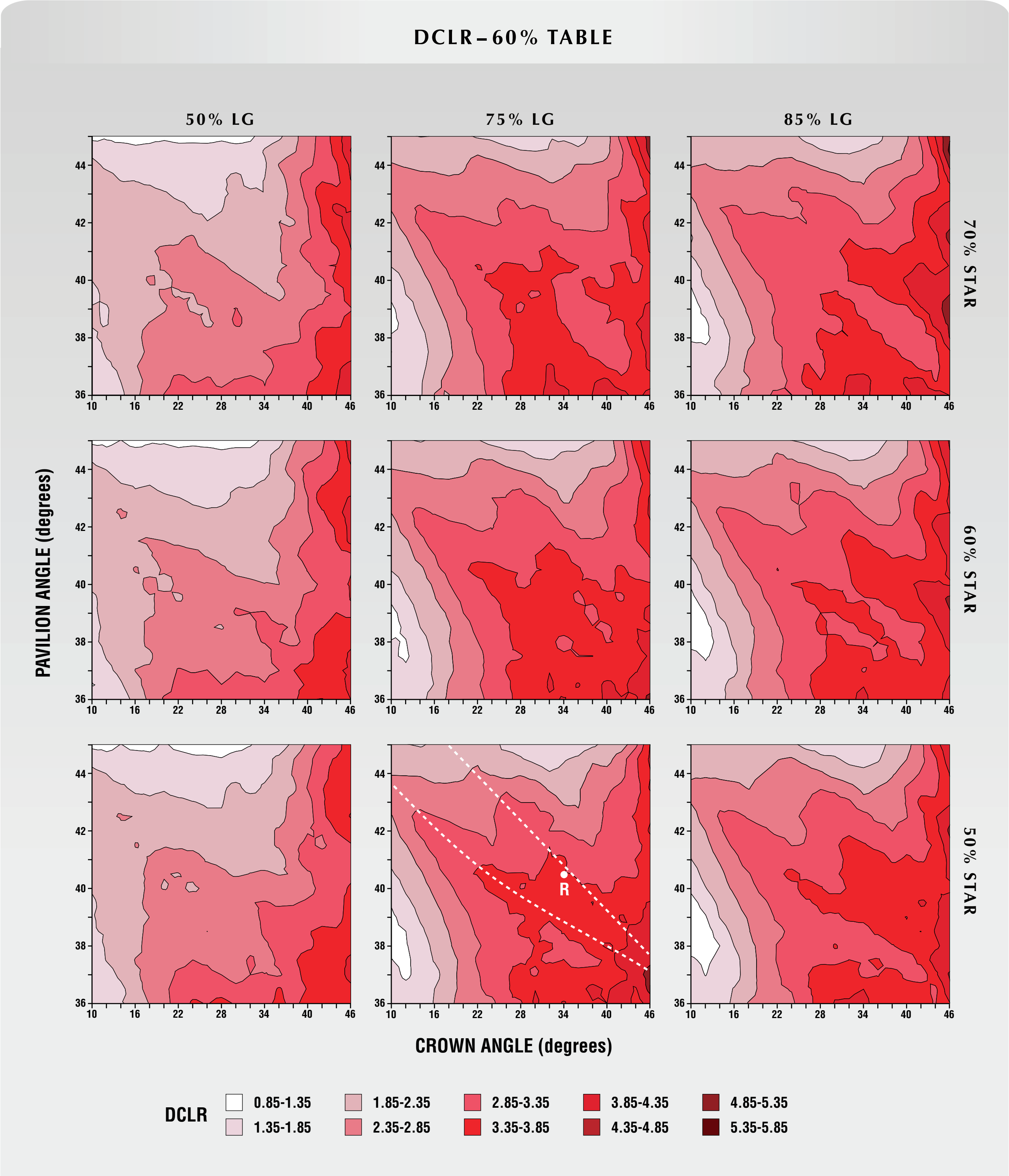


Figure 11. These nine contour plots show the variation in DCLR with changes in both crown and pavilion angles, for three values each of the star and lower-girdle (LG) facet lengths, at a table size of 60%. The DCLR surfaces are quite irregular, but they show that many proportion combinations yield above-average DCLR values, and others produce substantially lower values. The point marked "R" in the bottom center plot (75% lower-girdle length, 50% star length) is closest to our reference proportions (34° crown angle and 40.5° pavilion angle, although with a 60% table). The two dashed lines in this same plot indicate "ridges"—combinations of proportions that yield higher DCLR values than the proportions to either side. While each plot shows the detailed effects of varying crown and pavilion angles, comparison of the nine plots shows that star length and lower-girdle length also affect this metric.

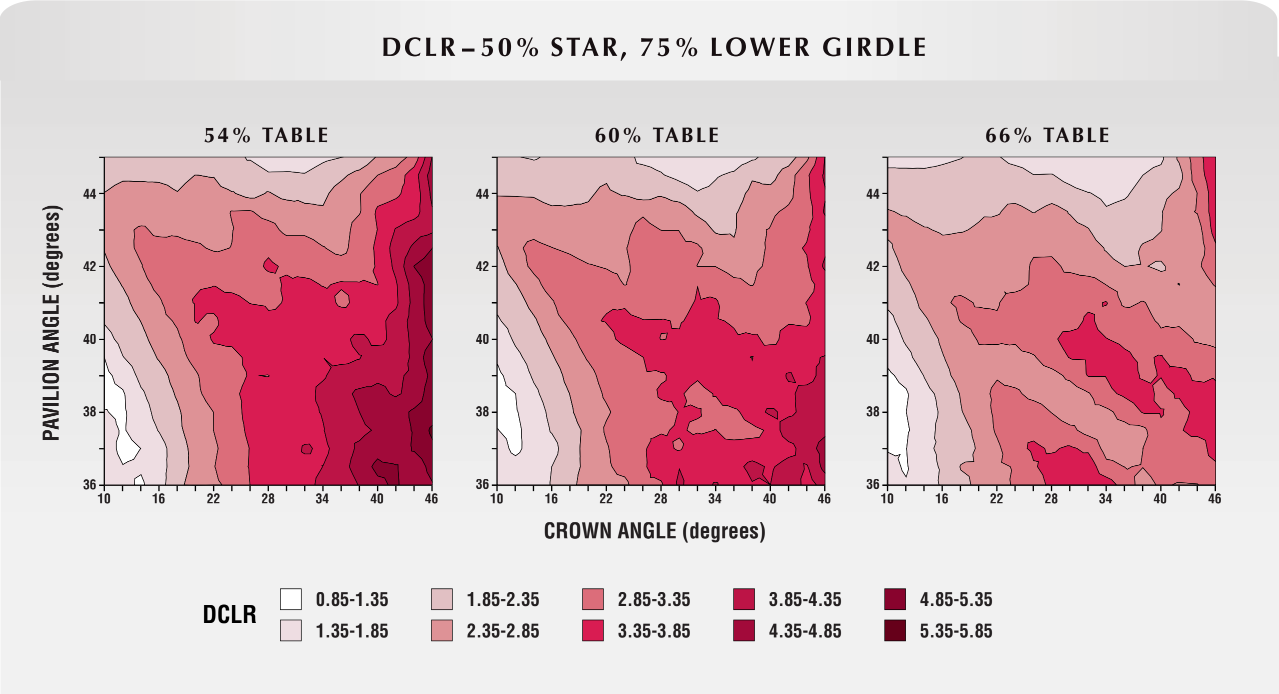


Figure 12. The possible combinations of crown angle and pavilion angle that yield above-average DCLR values greatly decrease as table size increases. In addition, the specific crown and pavilion angles that produce above-average DCLR values change toward higher crown angle and lower pavilion angle as table size becomes very large.

short or medium lengths for all these crown and pavilion angle combinations. Although steep pavilion angles are usually considered undesirable, the use of long lower girdle facets and small tables produces above-average DCLR values. Changing the star facet length for the same combinations of crown and pavilion angles had a less pronounced effect, as shown in figure 14.

Back to Brilliance: The Effect on WLR of Varying Star and Lower-Girdle Lengths. Recall that WLR values are much smaller than DCLR values; a change in WLR of 0.005 is discernable to trained observers (Hemphill et al., 1998). We analyzed data for sets of proportions (table size, crown angle, and pavilion angle) yielding high-average (0.280) and moderately low (0.265) WLR values when the star and lower-girdle facet lengths were at the reference proportions. Varying the length of the star or lower-girdle facets for these proportion combinations could increase WLR by 0.007, or decrease it by 0.015 (table 3). Many more proportion combinations led to decreases than to increases. Commercially common round brilliant proportions (see box B) showed smaller changes in the WLR value, whether increases or decreases, than more unusual proportions. The specific variations in star or lower-girdle facet length that produced the greatest increase in WLR depended strongly on the combination of crown angle, pavilion angle, and table size. For a broad, commercial range of crown angles and pavilion angles, with smaller tables (53%–57%), longer star facet lengths produced increases in WLR (see, e.g., figure 15).

DISCUSSION

DCLR and Proportions. Some proportion combinations that yield high DCLR values are not contiguous to one another, as shown in the contour plots (again, see figures 11–14). Thus, for some given values of two proportions, changes in the third proportion in a single direction may produce lower and then higher DCLR values. This variation in DCLR with different proportion combinations prevents the simple characterization of the "best" diamonds, in terms of fire, by evaluation of individual proportion parameters. Rather, it is the interaction between the proportions of the pavilion (pavilion angle and lower-girdle facet length) and those of the crown (table size, crown angle, and star facet length) that determines how light is dispersed by the round brilliant. Although certain generalizations may be made about the effects of a single proportion, in most cases there exist combinations of proportions that compensate to change DCLR. For example, large tables generally produce low DCLR, but with sufficiently high crown angles and shallower pavilion angles, a round brilliant with a table as large as

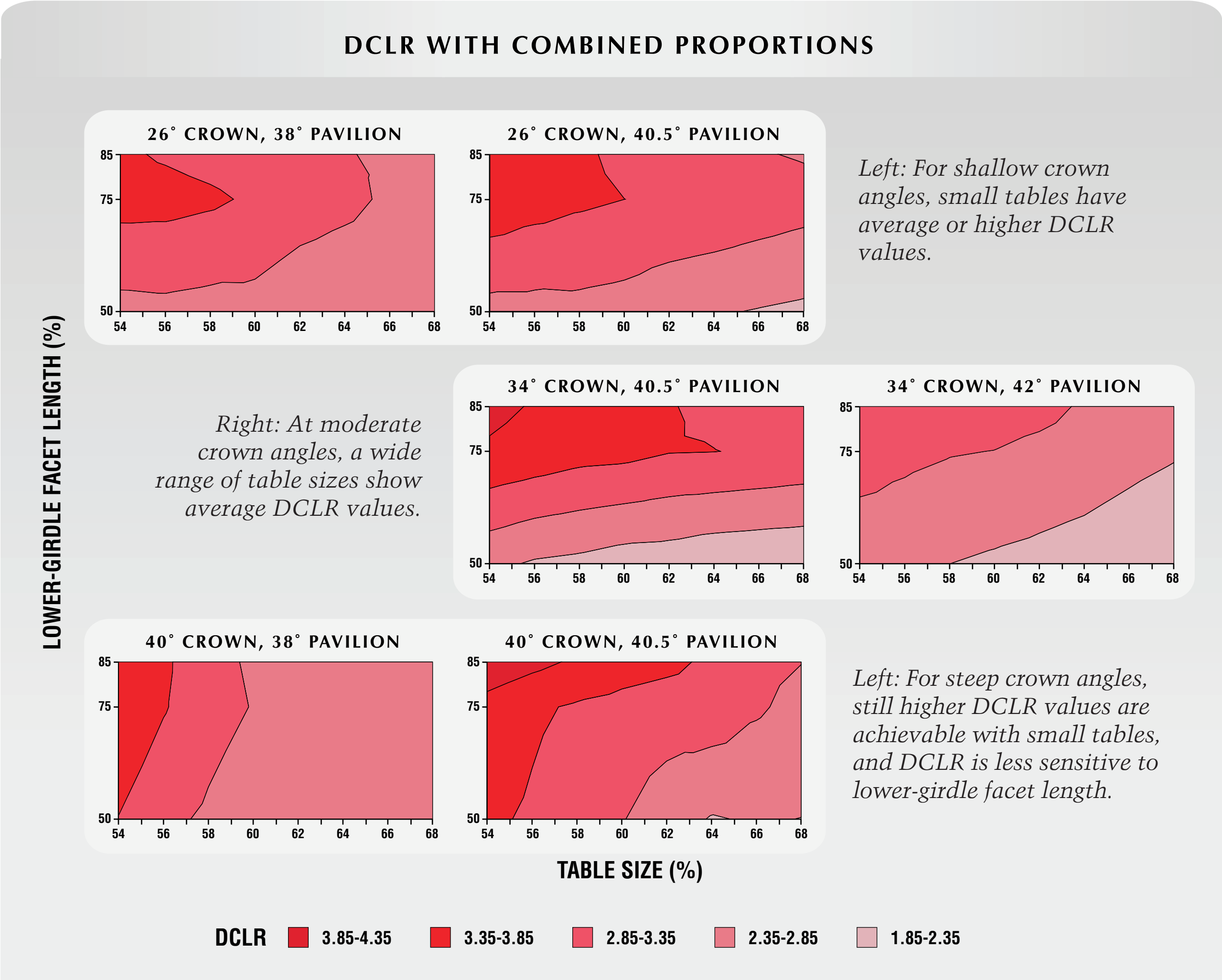


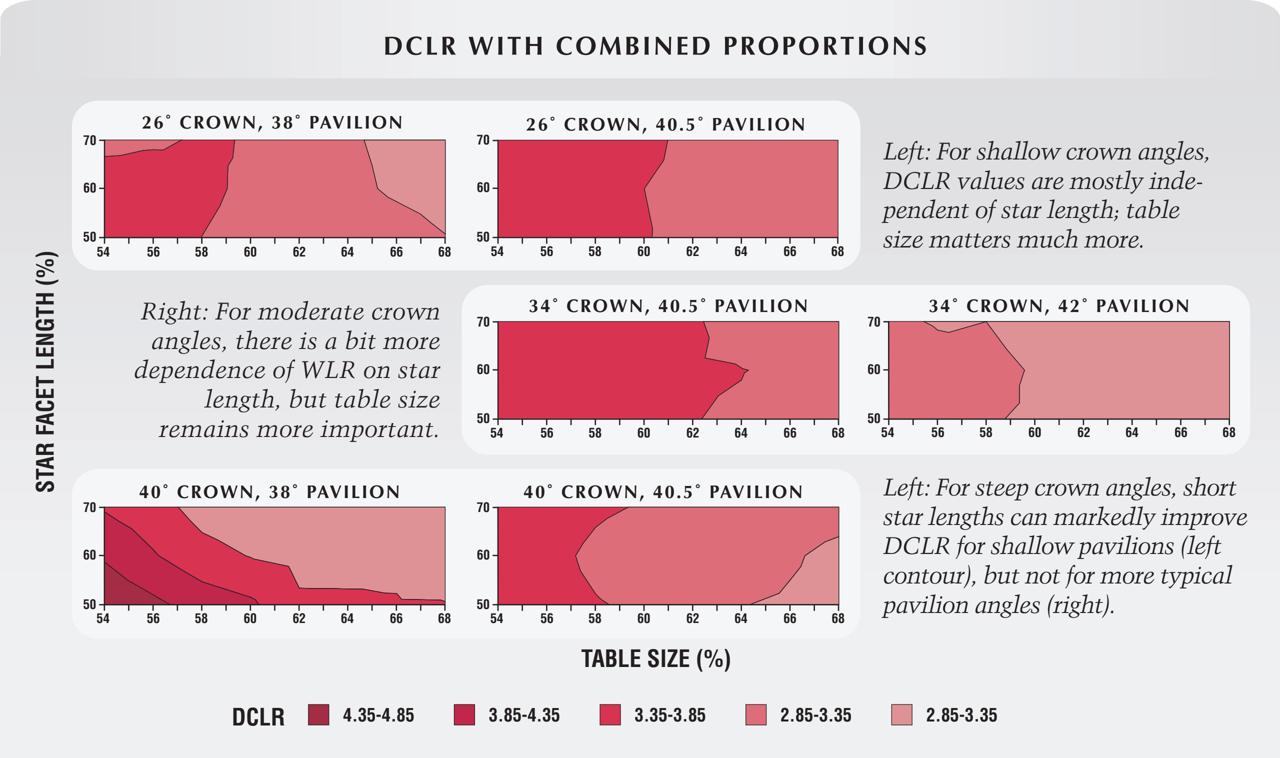
Figure 13. By grouping the contour plots, we can see how lower-girdle facet length, table size, and crown and pavilion angles work together to affect DCLR. Moderate to longer lower-girdle facet length generally increases DCLR. This effect is most pronounced at moderate pavilion angles.

66% can still yield an average or better DCLR value (see, e.g., diamond RD24 in table 2).

A short lower-girdle facet length of 50%, common early in the 20th century, greatly decreases DCLR and thus creates relatively poor fire (see, e.g., figure 11). Because Marcel Tolkowsky also chose this lower-girdle facet length, he made the reasonable—at that time—assumption that dispersion within the diamond could be neglected (see p. 56 of his 1919 treatise). Given the longer lower-girdle facets cut today, however, such an omission results in a poor approximation of how light moves within a round brilliant diamond.

Recall that fire is the visible extent of light dispersed into spectral colors. The type of lighting under which a round brilliant cut diamond is observed strongly affects the quantity and quality of colored light that emerges from it. With increasing distance, this colored light can spread out over a wide area; however, the eye can see only a fairly small area from one viewpoint. Thus, observation of fire depends strongly on where the eye is relative to the diamond, particularly how far away the observer is. Capturing all of the fire from the crown of a diamond required a multitude of observer positions, as if the diamond was being rocked. To achieve this, we used the same hemisphere of observations, and the same position-dependent weights for observations, for fire (DCLR) as we did for brilliance with the WLR metric.

However, we used a very different lighting condition for modeling fire than we used for modeling brilliance. For actual diamonds, diffuse lighting brings out brilliance and suppresses fire, while spot lighting does the opposite (again, see figure 1). By examining brilliance with fully diffuse light, and fire with a point source of light, we have explored the maximum extent to which the proportions of a



Left: For shallow crown angles, DCLR values are mostly independent of star length; table size matters much more.

Right: For moderate crown angles, there is a bit more dependence of WLR on star length, but table size remains more important.

Left: For steep crown angles, short star lengths can markedly improve DCLR for shallow pavilions (left contour), but not for more typical pavilion angles (right).

Figure 14. Changes in star facet length produced less change in DCLR values than was the case for lower-girdle facet length.

round brilliant can affect each of these appearance aspects. Every kind of lighting in the real world is some combination of diffuse light and directed (spot) light, and can be approximated in a computer model by combining fully diffuse and single-point sources.

Although three other teams of investigators (see Background on p. 175) modeled three-dimensional light movement in round brilliant cut diamonds, only two of these groups provided metrics for fire. Dodson (1978, 1979) offered three metrics (brilliance, fire, and "sparkliness"), and calculated their value for 120 proportion combinations of pavilion angle, crown angle (given as crown height), and table size. The MSU group ("Diamond cut study," 2001) offers three metrics (brilliance, fire, and a combination of these called Q for "quality"), and they calculate these metrics across a broad range of crown and pavilion angles for two table sizes (53% and 60%).

These researchers and our group use similar verbal definitions for *brilliance*, *fire*, and *scintillation*, but the exact metrics differ, particularly the ones for fire. The DCLR surfaces that we calculated as a function of five proportion parameters are quite irregular, more so than the WLR surfaces (again, see Hemphill et al., 1998). These multiple "peaks" imply that there are many combinations of parameters that yield equally fiery round brilliant diamonds, which is in general agreement with the results of Dodson and the MSU group, notwithstanding the differences in detail.

Dodson used "pseudo-diffuse illumination," and recorded his output on a virtual plane at a finite distance above his modeled diamond, after the spot technique of Rösch (1927). He then rotated the colored streak pattern, and evaluated fire by measuring the largest distance between observed red and blue streaks (which are actually circles, because the pattern is rotated 360°). Like Tolkowsky, Dodson tried to determine the widest spread of dispersed light from the diamond, rather than include, for example, the total number of flares and their intensities. Since this approach is so different from ours, it is not surprising that Dodson's numerical results show no particular correlation to our results. For instance, Dodson's most fiery diamond example had a 26.5° crown angle, a 43° pavilion angle, and a 60% table, which our model predicts would produce average fire (DCLR value of 2.9).

The MSU group calculates their fire metrics by finding the colored areas visible at the surface of the virtual diamond, seen from all directions within a 60° cone around the vertical axis. For each patch of fire, they compute the color difference from white (ΔE or ΔH; see again C.I.E., 1963), multiply it by the square root of the area the patch covers, and sum these values. Their fire metric MF 30 averages the calculated

TABLE 3. Largest WLR changes from variation of both star and lower-girdle facet lengths.[a]

Table size (%)	Crown angle (°)	Pavilion angle (°)	Highest WLR found	At star facet length (%)	And at lower girdle-facet length (%)	Lowest WLR found	At star facet length (%)	And at lower girdle-facet length (%)
Sets of proportions that yield the high-average WLR value of 0.280[b]								
54	*31*	*38.5*	*0.284*	*68*	*75*	*0.273*	*30*	*85*
54	35	40.5	0.281	54	75	0.273	74	50
54	33	42.5	0.283	66	50	0.275	30	85
59	*30*	*43.0*	*0.287*	*74*	*50*	*0.273*	*30*	*85*
59	32	41.5	0.281	72	50	0.273	30	85
60	*29*	*43.0*	*0.287*	*74*	*50*	*0.275*	*30*	*85*
60	*26*	*38.0*	*0.283*	*34*	*75*	*0.268*	*74*	*85*
60	*30*	*39.75*	*0.283*	*62*	*75*	*0.271*	*30*	*85*
62	29	42.0	0.285	74	50	0.273	30	85
62	*28*	*42.75*	*0.287*	*74*	*50*	*0.274*	*30*	*85*
Sets of proportions that yield the moderately low WLR value of 0.265[b]								
54	*38*	*38.75*	*0.266*	*58*	*75*	*0.260*	*30*	*50*
54	37	42.25	0.265	56	75	0.259	30	85
57	38	41.75	0.265	52	75	0.257	74	85
59	59	40.5	0.265	50	75	0.259	74	85
59	59	42.0	0.265	48	75	0.257	74	85
60	*60*	*38.5*	*0.266*	*58*	*75*	*0.262*	*30*	*50*
60	60	40.5	0.265	42	75	0.259	74	85
60	60	42.0	0.265	48	75	0.257	74	85
62	*62*	*38.25*	*0.268*	*68*	*75*	*0.255*	*30*	*85*
62	62	41.75	0.266	46	75	0.258	74	85

[a] *Proportions that are not commonly seen in the trade are italicized here.*

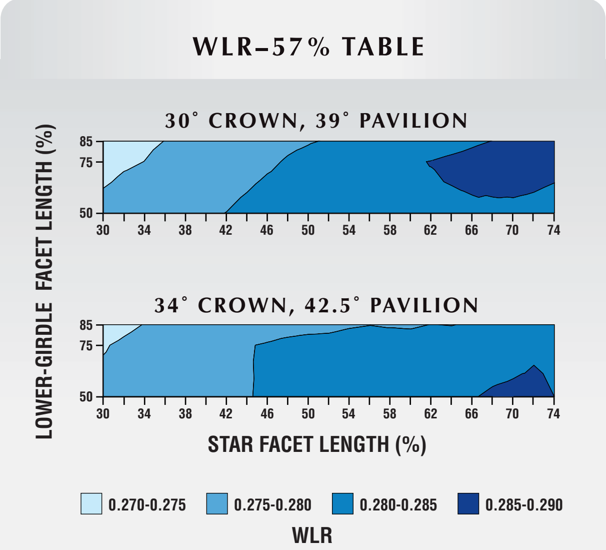
[b] *Each example yielded the WLR value of 0.280 (or 0.265) with a star length of 50% and lower-girdle length of 75%.*

fire from a combination of diffuse and directed light.

Their results for 53% and 60% table sizes (again, see "Diamond cut study," 2001) show the highest fire for diamonds with very shallow crown angles (less than 20°) and very steep pavilion angles (45°–50°). They found the least fire for diamonds with typical crown angles (30°–36°) and steep pavilion angles (greater than 43°) at both table sizes. In contrast, we found that diamonds with steeper crown angles (greater than 36°) show high fire across the range of pavilion angles we considered (36°–45°); the lowest fire predicted by our model is produced by diamonds with very shallow crown angles and either very shallow or very steep pavilion angles (again, see figure 11).

Additional WLR Results. We stated in 1998 that differences in WLR values as small as 0.005 could be discerned by trained observers in actual diamonds under controlled conditions. Varying the length of the star or lower-girdle facets (or both) can increase WLR by slightly more than this amount, or decrease it by three times this amount. Since these facets are polished at the end of the manufacturing process, judicious choices for their lengths can help improve the appearance of a round brilliant when the shape of the rough prompts choices of crown angle, pavilion angle, and/or table size that typically produce only moderate values of WLR. Note that none of the major current grading systems includes

Figure 15. As seen in these plots for two different sets of crown and pavilion angles (with a constant table size of 57%), varying the lengths of the star and lower-girdle facets can have a significant impact on WLR.



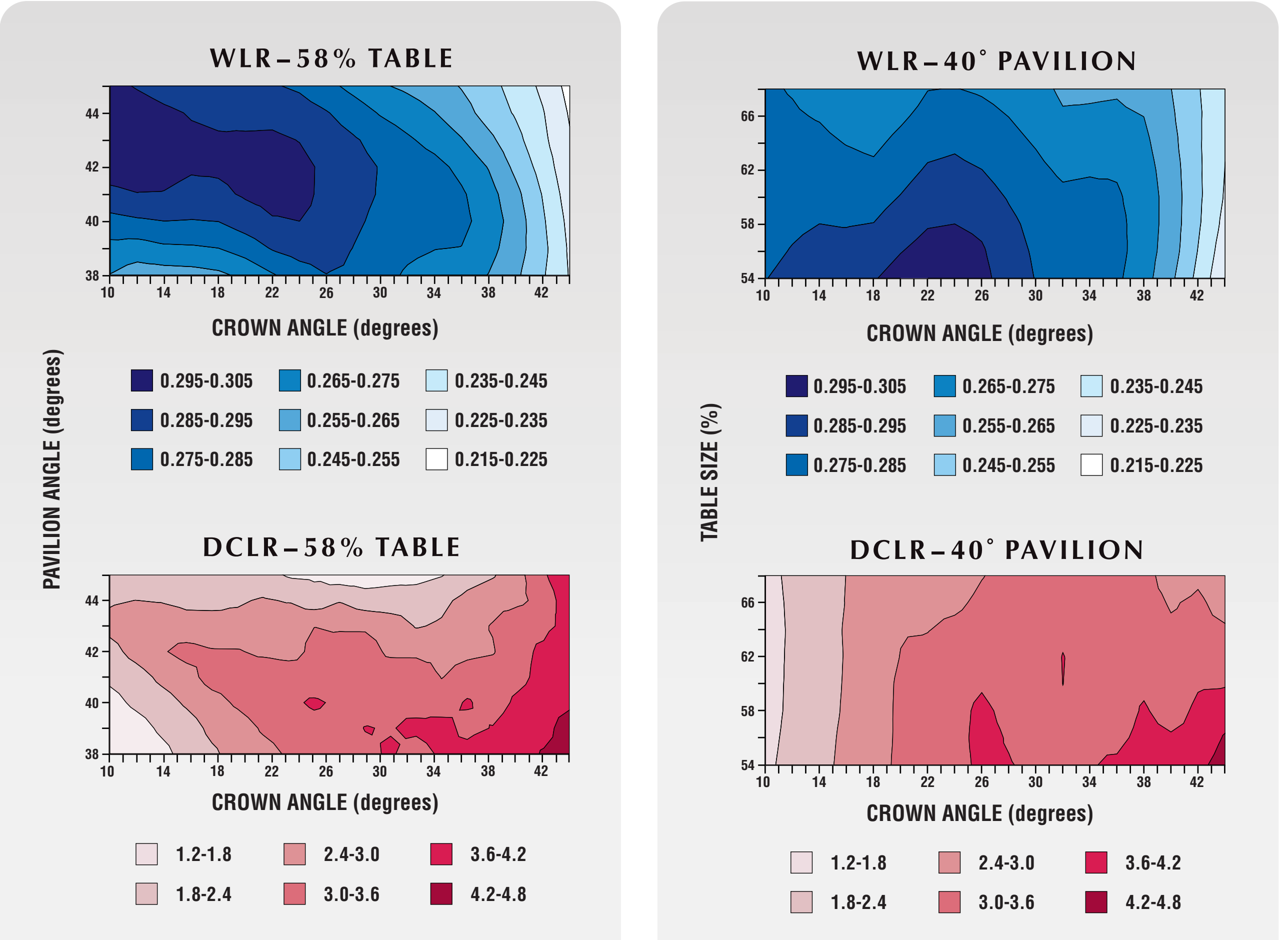


Figure 16. Left: These contour plots show WLR and DCLR across the same ranges of crown and pavilion angle for a 58% table, 50% star facet length, and 75% lower-girdle facet length. Although there is no overlap in the proportions that produce the highest values of each of these metrics, many proportion combinations produce average (0.270–0.280 WLR, 2.8–3.5 DCLR) or higher values of both metrics. Right: These contour plots show WLR and DCLR for a range of crown angles and table sizes for a pavilion angle of 40°. While the value of both metrics decreases as table size increases, many crown angles produce above-average values of both for table sizes up to 58%.

the length of star or lower-girdle facets in their analysis of cut.

Interplay of Brilliance and Fire. It is difficult to say at this time whether brilliance or fire has more of an impact on the overall appearance of a round brilliant diamond. Nevertheless, with the results modeled for both brilliance (WLR) and fire (DCLR), we are able to identify proportion combinations that produce above-average values of both appearance aspects. In figure 16, we show two cross-sections through proportion space with WLR contoured in shades of blue and DCLR contoured in shades of red. Each metric can be compared to the mean values for our population of more than 67,000 actual diamonds (see box B in this article and the corresponding box B in Hemphill et al., 1998).

Figure 16 (left) shows variations in WLR and DCLR with variations in crown angle and pavilion angle for the commercially important table size of 58% and reference values for the other properties. The highest WLR values appear at shallow crown angles and moderate-to-steep pavilion angles. However, the WLR values are still above average for many crown and pavilion angle combinations. The highest DCLR values occur in a small area at the opposite corner of the plot (at a steep crown angle and a shallow pavilion angle), but the range of proportions that show above-average fire is quite large. Although there is no overlap in the areas that produce the *highest* values shown for each metric, there is considerable overlap in the proportion combinations that produce average or higher values of both at this table size. For example, crown angles from 24° to 32° combined with pavilion angles from 38° to 42° yield WLR above 0.270 and DCLR of 3.0 or

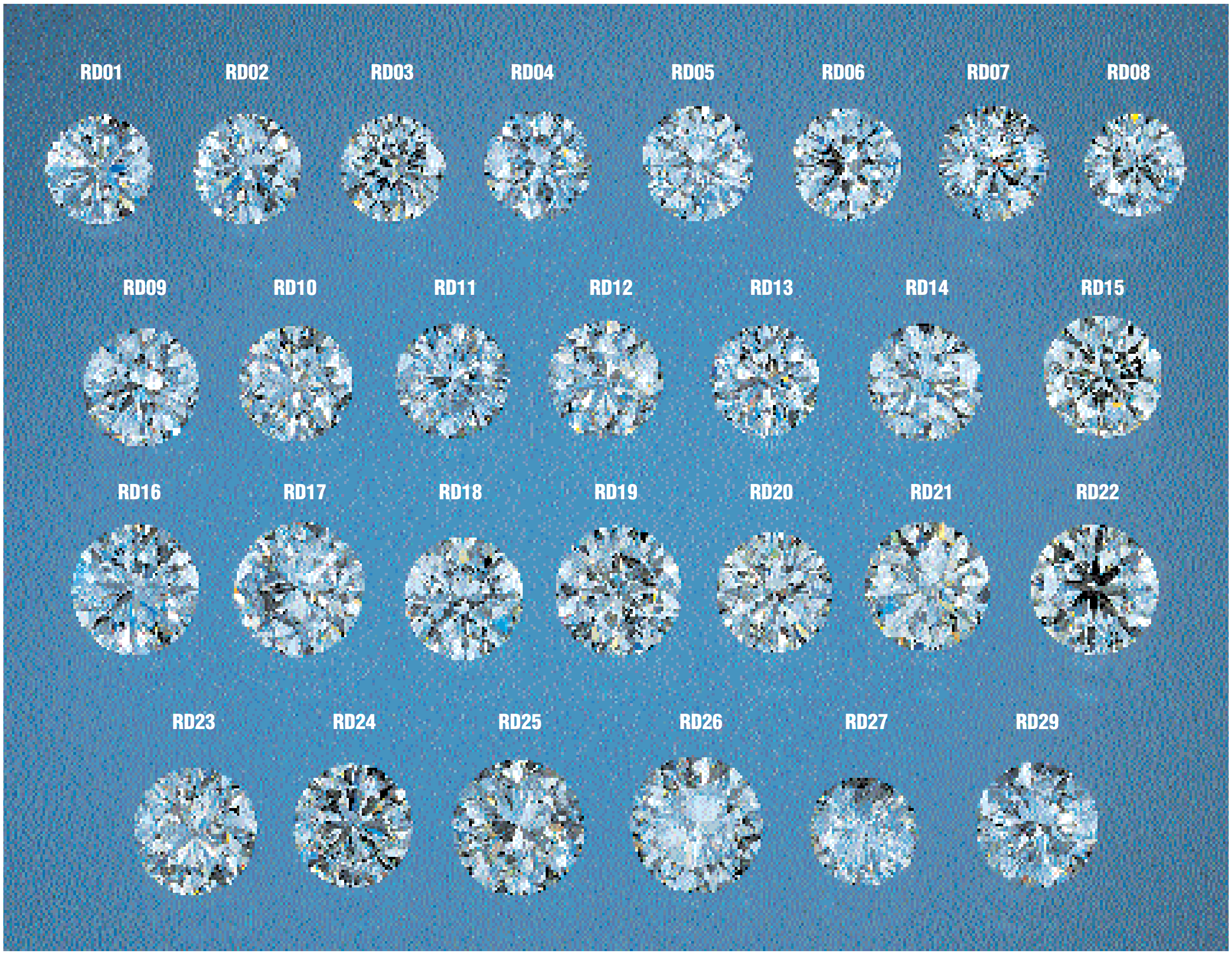


Figure 17. These 28 diamonds (0.44–0.89 ct) were used to examine the brilliance and fire associated with different sets of proportions. Some sets of proportions in this group are decidedly non-commercial (again, see table 2). Photo by Harold & Erica Van Pelt.

more; and crown angles from 20° to 36° accompanied by pavilion angles of 40° to 41° yield the same results.

Figure 16 (right) illustrates the dependence of WLR and DCLR on variations in crown angle and table size for a commercially relevant pavilion angle of 40° and reference values for the other parameters. There is one area of overlap where both WLR and DCLR are well above average, at crown angles of 26° to 28° and table sizes less than 58%. These smaller tables yield high values of DCLR at crown angles greater than 35°, but corresponding WLR values decrease steeply to below average as crown angle increases beyond 36°. However, there is a broad area (crown angles from 26° to 36° and table sizes up to 62%) that yields above-average values for both metrics.

The specific combinations of crown and pavilion angles that produce above-average values for each metric vary with table size, star length, and lower-girdle length. For example, a lower-girdle facet length of 50% greatly reduces DCLR, but it can produce small increases in WLR; similarly, star facet lengths greater than 50% can increase WLR but will have a variable effect on DCLR, depending on the crown angle (again, see figures 14 and 15). Note also that there are far more combinations of crown angle and pavilion angle that work well with small tables (less than 55%) than with large tables (66% or more).

In some areas of proportion space, small changes in one or more proportions lead to fairly large changes in the WLR and DCLR values. If we use the analogy of land surfaces, these regions of proportion space are like steep mountainsides. Other areas are more like plateaus: regions where fairly large changes in proportions do not have a significant effect on WLR or DCLR values. The typical commercial range of proportions (again, see box B) lies

Figure 18. Although these two diamonds (RD19, left; RD21, right—see table 2) have different crown angles (29.2° and 35.8°), they display comparable amounts of fire with spot lighting. Photo by Harold & Erica Van Pelt.

Figure 19. As reported in table 2, the diamond on the far left (RD24) has a large (66%), table while the other three (RD01, RD23, and RD13) have much smaller tables (52% to 54%). However, all have comparable DCLR values. Photo by Harold & Erica Van Pelt.

within such a plateau. Thus, the cutter has a broad latitude for choices of proportions without risking a catastrophic effect on appearance.

Comparing Model Results to Real Diamonds. The ultimate verification of any model of diamond appearance is a comparison with actual diamonds cut to specific proportion combinations. We assembled the group of 28 round brilliant cut diamonds (0.44–0.89 ct) shown in figure 17; their proportions, color and clarity grades, and calculated WLR and DCLR values are given in table 2. As noted below, various round brilliants in this group (again, see figure 1) both confirm the fitness of our model and defy a few common beliefs in the trade about the relationship between proportions and appearance.

There are many combinations of crown angle, table size, and pavilion angle that produce average or higher DCLR values. This range of proportions is part of the "broad plateau" described above. For instance, diamond RD19 has a rather shallow crown angle, whereas RD21 has a fairly steep crown angle; yet both show considerable fire (figure 18). In both cases, the model indicates that this fire results from the combination of the diamond's particular pavilion angle with the crown angle and table size (as well as additional proportions).

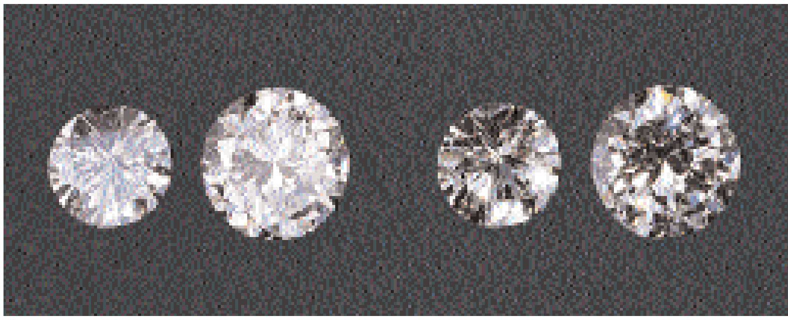
Diamond RD13, with a 52% table, gave the highest DCLR value (4.01). The next two highest DCLR values (3.97 and 3.92) were calculated for diamonds RD01 and RD23, which also have small tables (54%). However, one diamond with a large (66%) table, RD24, yielded a high average DCLR value of 3.23 (figure 19).

We also compared our model results for the combination of WLR and DCLR (again, see figure 1 and table 2) with the appearances of actual diamonds cut to these proportions. For steep pavilion angles, there were no crown angles that yielded particularly high values of both WLR and DCLR; RD25 is both dark and lacking in fire despite its steep crown angle (figure 20). Also shown is RD27, with a much deeper pavilion angle and an extremely shallow crown; it shows little brilliance or fire (although MSU results would suggest it to be quite fiery).

CONCLUSIONS

Our overall research goal is to understand why a round brilliant cut diamond looks the way it does. Its appearance is a complex mixture of the effects of various lighting and observing conditions, the specific characteristics of each diamond, and the interpretation by the human visual system of the overall pattern of light shown by the diamond. For our study thus far, we found that different lighting conditions are needed to bring out the maximum brilliance (totally diffused light) and fire (directed light). We made simplifying assumptions about the diamond

Figure 20. These two diamonds, with 50.7° (RD27, left image in each pair) and 41.8° (RD25, images at the right) pavilion angles, are neither bright (left pair, with diffuse lighting) nor fiery (right pair, with spot lighting). Composite of photos by Harold & Erica Van Pelt.



(e.g., colorless and perfectly symmetric, among other conditions) and the observing conditions (e.g., that a weighted average of viewing positions corresponds to the observation of a diamond being "rocked").

Through this research, we have gained a much better understanding of some of the key factors that govern the appearance of a round brilliant cut diamond, particularly how the proportions of that diamond (expressed as eight independent proportion parameters) affect its brilliance and fire. In this article, we showed that VFI diagrams (our model's graphic output of dispersed light from virtual diamonds) match the pattern of chromatic flares from actual diamonds. We presented our metric for fire, dispersed colored light return (DCLR), which we have computed for more than 26,000 combinations of the proportion parameters. We also showed that star and lower-girdle facet lengths could have a noticeable effect on WLR, our metric for brilliance.

Every facet matters in a round brilliant diamond. In general, DCLR is higher for smaller table sizes and larger crown angles, but at least three other parameters also are important: pavilion angle, star facet length, and lower-girdle facet length. Our modeling results indicated that a diamond with a shallow crown angle or a large table could still display higher-than-average fire if combined with the right pavilion angle, star facet length, and lower-girdle facet length. The relative appearances of our 28 actual diamonds with specific proportions confirmed these predictions.

When analyzing fire, both lighting and observing conditions strongly affect the analysis of fire. We used two distinct lighting conditions: fully diffused light to analyze brilliance (WLR), and a point-light source (i.e., a modeled spotlight) to analyze fire (DCLR). These different idealized lighting conditions are useful for determining the maximum extent of these appearance aspects. By its nature, fire spreads out as it emerges from a diamond, so multiple views are needed to observe it. For this reason, we used the same weighted hemisphere of observation that we used for analyzing brilliance. Other researchers have investigated fire, but their metrics and their results are substantially different from ours.

We have not found a "bull's-eye" of "best" proportions for either DCLR or WLR. Rather, both metrics show a complex dependence on proportion combinations, with many sets of proportions that yield similar values. Comparison of these two metrics shows that some proportion combinations yield the highest DCLR values, and others yield the highest WLR values, but in general these proportions do not overlap. Nevertheless, there are many choices of diamond proportions that give average or higher values for both parameters. Star and lower-girdle facet lengths can increase or decrease the calculated value of either metric, but the proportion variations that increase one metric may decrease the other.

Although our research is not yet complete, we believe that this understanding as to how cut proportions work together may bring a new degree of freedom to diamond manufacturers. By offering a wider range of cutting choices, this information may help manufacturers produce round brilliant diamonds with both above-average performance and potentially higher weight yields from the rough. Although this approach provides more options for cutting rough efficiently, it will also require more precision on the part of manufacturers. In addition, we feel the detailed information gathered to date on the levels of brilliance and fire provided by different proportion combinations can potentially serve as a basis for a more in-depth cut evaluation system.

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